Modeling the Effects of Elasticity

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1. Introduction

The main purpose of this study is to examine the effects of elasticity on the free internal and forced external modes. Our methodology is based on the analysis of the linear solutions of selected systems with varying degrees of elasticity. These systems are the nonhydrostatic fully-compressible (FC), nonhydrostatic unified (UN), quasi-hydrostatic (QH), nonhydrostatic pseudo-incompressible (PI) and nonhydrostatic anelastic (AN) systems. The FC used in many nonhydrostatic models (e.g. Satoh et al., 2008 and others) is fully-elastic and permits acoustic waves. The UN of Arakawa and Konor (2009) and the QH provide elasticity, but filter the vertically propagating acoustic waves of all scales. The PI and AN are anelastic and exclude all effects of elasticity to filter all types of acoustic waves (e.g. Smolarkiewicz et al., 2001 and others).

2. Results

Arakawa and Konor (2009) present a normal mode analysis on an *f*-plane without the quasigeostrophic approximation and on a midlatitude β-plane with the quasigeostrophic approximation. Fig. 1 shows the dispersion of quasigeostrophic modes obtained by the elastic systems, i.e. FC, UN and QH, and the anelastic systems, i.e. PI and AN. The most important difference between the results of the elastic and anelastic systems appears in the barotropic modes. The elastic systems produce the "compressible" Rossby waves that have a bounded retrogression speed – the speed is defined as $c \equiv \nu/k$ – (Fig. 1a) while the anelastic systems produce the "incompressible" Rossby waves that have an unbounded retrogression speed given by $c \equiv -\beta/k^2$ (Fig. 1b). Thus, the anelastic systems may produce ultra long waves with a very high retrogression speed that is not seen in Nature.

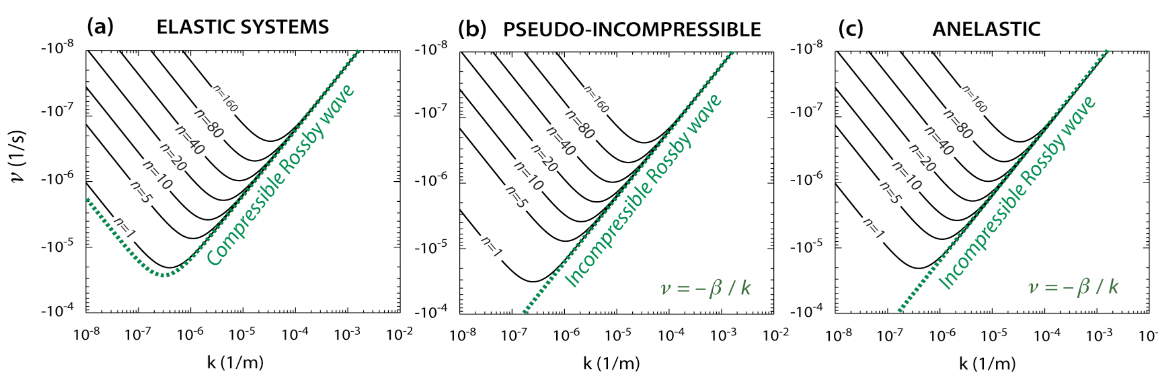

Fig. 1. Frequencies (ν's) of normal modes on a midlatitude β-plane with the quasigeostrophic approximation as functions of horizontal wavenumber *k* for (a) the fully compressible, unified, and quasi-hydrostatic, (b) pseudo-incompressible, and (c) anelastic systems. In these figures, *n* is the vertical wavenumber of the baroclinic modes. Green dashed line shows the dispersion of the barotropic modes (Rossby waves).

To examine the effects of elasticity on the forced modes, we first define the elasticity ε as

$$\varepsilon \equiv (1/VMF)(\partial\rho/\partial t) = (HMF/VMF) + 1, \tag{1}$$

where VMF [$\equiv \partial(\rho w)/\partial z$] is the convergence of vertical mass flux, ρ is the density, w is the vertical velocity, HMF [$\equiv \nabla_H \cdot (\rho \mathbf{v})$] is the convergence of horizontal mass flux, ∇_H is the horizontal del operator, and **v** is the horizontal velocity. Fig. 2 shows the elasticity as a function of the vertical wavenumber n for the forced modes with the horizontal wavenumbers between k=10^{-5} m^{-1} and 10^{-7} m^{-1} on an f-plane obtained by the elastic systems, i.e. FC, UN and QH, (Fig. 2a) and the anelastic system (Fig. 2b). Note that the vertical wavenumber is not an integer for the forced modes. With the elastic systems, the horizontal mass convergence (or divergence) associated with the very deep modes is partially compensated by the vertical mass divergence (or convergence) because the elasticity absorbs a large portion of the horizontal mass convergence (or divergence). The singularity appears in Fig. 1a because the vertical mass convergence changes sign for the deep modes. With the anelastic system, the horizontal mass convergence (or divergence) associated with all modes is completely compensated by the vertical mass divergence (or convergence).

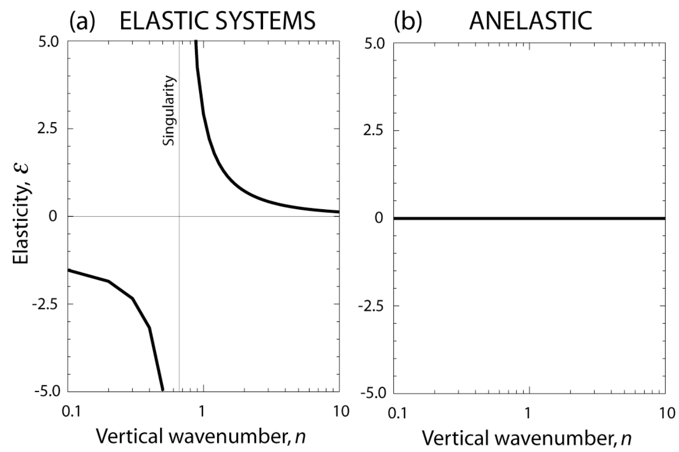

Fig. 2. Elasticity of forced modes. See text for explanation.

4. Conclusions

From the results of analyses, it can be concluded that elasticity is needed for accurate simulations of the dispersion (and retrogression speed) of ultra long waves. The effects of elasticity is also important for very deep forced modes. If an anelastic system is used in a global model, an ad hoc elasticity needs to be added in the discretization step to avoid errors in ultra long waves and deep forced modes.

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