On the Development of a Vector Vorticity Model Based on an Icosahedral Grid

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1. Introduction

We discuss the computational design of an anelastic vector-vorticity dynamical core (VVDC) implemented on a spherical icosahedral grid. The prognostic variables for this model are the horizontal and vertical components of vorticity. The horizontal vorticity is predicted at all levels. To enforce the non divergence of the 3D vorticity vector the vertical component of vorticity is predicted only in the top layer and diagnosed throughout the remainder of a model column. The vertical velocity is obtained as the solution of a three-dimensional elliptic equation. This work is an extension to the sphere of the model originally designed for a Cartesian grid by Jung and Arakawa (2008).

2. Model description

We define the three-dimensional vorticity and velocity in terms of their horizontal and vertical components $\boldsymbol{\omega} \equiv \boldsymbol{\eta} + \zeta\mathbf{k}$ and $\mathbf{V} \equiv \mathbf{v} + w\mathbf{k}$. The prognostic equations for the horizontal and vertical components of vorticity are given by

$$\begin{aligned} \frac{\partial \boldsymbol{\eta}}{\partial t} &= -\boldsymbol{\eta}(\nabla \cdot \mathbf{v}) - (\mathbf{v} \cdot \nabla)\boldsymbol{\eta} - \frac{\partial}{\partial z}(\boldsymbol{\eta} w) + (\boldsymbol{\eta} \cdot \nabla)\mathbf{v} + \zeta \frac{\partial \mathbf{v}}{\partial z} - \mathbf{k} \times \nabla B \\ \frac{\partial \zeta}{\partial t} &= -\nabla \cdot (\zeta \mathbf{v}) - \frac{\partial}{\partial z}(\zeta w) + \boldsymbol{\eta} \cdot \nabla w + \zeta \frac{\partial w}{\partial z} \end{aligned} \tag{1}$$

where B represents buoyancy. Note that the pressure gradient term has been eliminated from these equations. The thermodynamic equation is given by

$$\frac{\partial(\rho_0 \theta)}{\partial t} = -\nabla \cdot (\rho_0 \theta \mathbf{v}) - \frac{\partial}{\partial z}(\rho_0 \theta w) + \frac{\rho_0 Q}{c_p \pi} \tag{2}$$

where $\rho_0 \equiv \rho_0(z)$ is a horizontally uniform density, and Q is the heating per unit mass.

The vertical velocity is determined by solving an elliptic equation of the form

$$\nabla^2 w + \frac{\partial}{\partial z}\left[\frac{1}{\rho_0}\frac{\partial}{\partial z}(\rho_0 w)\right] = -\mathbf{k} \cdot \nabla \times \boldsymbol{\eta} \tag{3}$$

with the boundary conditions $w_S = w_T = 0$ at the lower and upper boundaries.

The horizontal structure of the model is based on an icosahedral grid, Fig. 1. The grid is constructed from an icosahedron through recursive bisection and subdivision. This provides almost homogeneous discretization and quasi-isotropic resolution over the sphere. The horizontal component of vorticity is defined at cell edges, the vertical component of vorticity is defined at cell corners, and potential temperature is defined at cell centers.

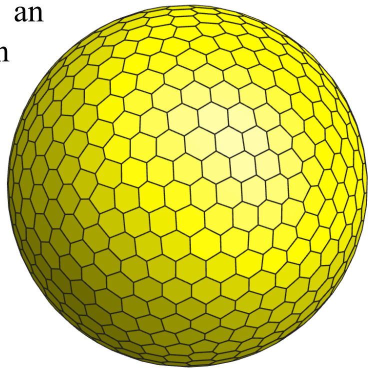

Figure 1. An Icosahedral grid with 642 cells

3. Numerical Results

To test the model we have performed a buoyant warm bubble experiment. For computational expediency the Earth's radius is reduced to 3633 m. This allows for cells with a 250 m horizontal extend using a global grid with 10242 cells. The model top is placed at 15000 m. With 100 model layers the aspect ratio of the cells in nearly unity. For these tests the Earth is not rotating.

The initial potential temperature is given by $\theta = \theta_0 + \theta'$ where $\theta_0 = 300K$ and

$$\theta' = \mathrm{Max}\left\{0, 1 - \left(\frac{r}{r_1}\right)^2 - \left(\frac{z - z_0}{z_1}\right)^2\right\} \quad (4)$$

where $r_1 = 2500\mathrm{m}$, $z_0 = 4000\mathrm{m}$ and $z_1 = 2000\mathrm{m}$. The initial vorticity is given by $\boldsymbol{\eta} \equiv 0$ and $\zeta \equiv 0$ .Fig 2 shows cross sections of the time evolution of the rising bubble.

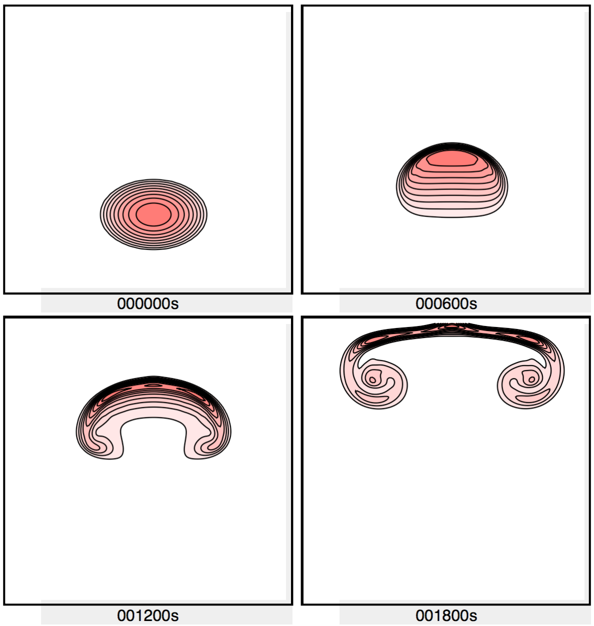

Figure 2. Rising buoyant bubble. The contour interval is 0.1 K

Acknowledgements

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References

Jung, J.-H., and A. Arakawa, 2008: A Three-Dimensional Anelastic Model Based on the Vorticity Equation. *Mon. Wea. Rev.*, **136**, 276-294.