Conservation of total energy in nonhydrostatic solvers

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1. Introduction

Conservation of total energy is rarely enforced in nonhydrostatic numerical models. Sometimes this is because an effect is negligible: for example, dissipative heating (the increase in internal energy that occurs when kinetic energy is dissipated) is very small in most weather systems. In other instances, an effect is not well understood: for example, when water drops fall through air there is a cooling effect on the air (even in the absence of evaporation) as water-drops warm to the ambient temperature. These two effects – dissipative heating and hydrometeor-fallout cooling – that are necessary for total-energy conservation have been included in CM1 (http://www.mmm.ucar.edu/people/bryan/cm1/), a nonhydrostatic model used for cloud research at NCAR, as described below.

2. Dissipative Heating

For viscous terms in the momentum equation of form $\partial u_i/\partial t = \ldots + \partial\tau_{ij}/\partial x_j$ then conservation of total energy requires a term in the temperature equation of form

$$c_v \frac{\partial T}{\partial t} = \ldots + \tau_{ij}\frac{\partial u_i}{\partial x_j}. \tag{1}$$

Here, u_i is velocity, T is temperature, τ_{ij} is viscous stress, and c_v is the specific heat of air at constant volume. Dissipative heating is often negligible, but it can be substantial (> 10 K h^{-1}) near the surface in strong wind conditions.

3. Hydrometeor-Fallout Cooling

The temperature tendency due to transport of internal energy by falling hydrometeors has the form

$$c_v \frac{\partial T}{\partial t} = \ldots + c_l r_l v_l \frac{\partial T}{\partial z}, \tag{2}$$

where c_l is the specific heat of water, r_l is mixing ratio of water, and v_l is the fall velocity of water. Because $\partial T/\partial z$ is typically negative in the atmosphere, the right side of (2) is typically negative (i.e., the air cools). Using values for heavily precipitating convective systems, hydrometeor-fallout cooling can have a magnitude of order 5 K h^{-1}.

4. Effect on Tropical Cyclone Intensity

To test how these processes affect organized convective systems, I have been conducting simulations of various types of convective systems. This presentation will focus on idealized simulations of tropical cyclones (TCs). The following simulations use the axisymmetric version of CM1 using the initial conditions, resolution, and physical parameterizations from Bryan and Rotunno (2009).

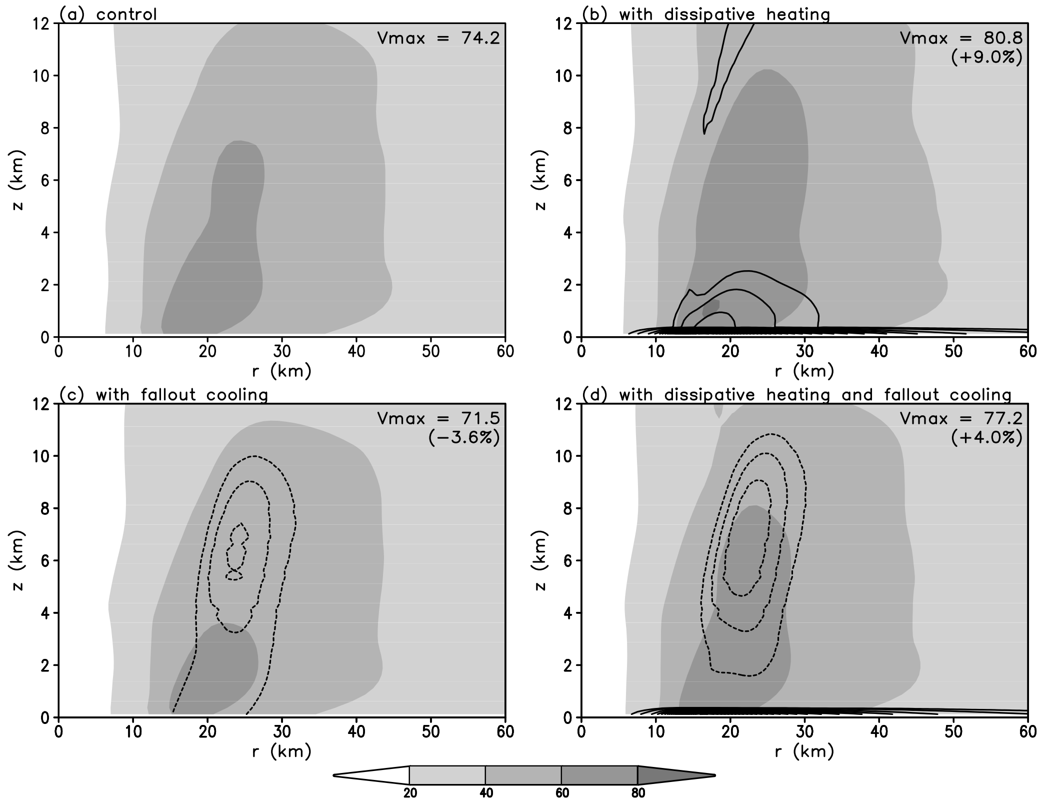

FIG. 1: Azimuthal velocity (shaded, m s^{-1}) and temperature tendency (contours every 1 K h^{-1}, negative values dashed) from axisymmetric simulations of a tropical cyclone: a) control, b) with Eqn. (1) included, c) with Eqn. (2) included, and d) with both Eqns. (1) and (2) included.

Fig. 1 shows output after 8 days of integration, where the contours are the temperature tendencies from Eqns. (1)–(2). Dissipative heating increases TC intensity in CM1 by ∼10% (Fig. 1b), consistent with the results by Bister and Emanuel (1998). The fallout cooling term decreases TC intensity by ∼4% (Fig. 1c). When both terms are included their effects almost cancel each other, although in this case there is a slight increase in TC intensity (Fig. 1d).

These energy-conserving terms have similar magnitude to the uncertainty from microphysical schemes and surface exchange coefficients. Hence, it is probably reasonable to exclude them from NWP models. Nevertheless, these effects should be included in numerical modeling systems in which exact conservation of total-energy is desirable, such as for long-term climate simulations.

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References

Bister, M., and K. A. Emanuel, 1998: Dissipative heating and hurricane intensity. *Meteor. Atmos. Phys.* **65,** 233–240.

Bryan, G. H., and R. Rotunno, 2009: The maximum intensity of tropical cyclones in axisymmetric numerical model simulations, *Mon. Wea. Rev.,* **137,** 1770–1789.